Symposium on Probability and Stochastic Processes
CIMAT, november 20-24

Abstracts

Courses

- **Govind Menon (Brown University)**

Title: *The deep linear network and random matrix theory*

Abstract: The deep linear network is a matrix model for deep learning that was proposed a few years ago by the computer scientists Nadav Cohen, Sanjeev Arora and Elad Hazan. It is a phenomenological model that captures some of the features of the gradient flows used to train neural networks. It has now become apparent that the model has very interesting mathematical structure; in particular, it has an explicit Riemannian geometry and several features that are reminiscent of random matrix theory, including exact formulas for volume forms, and interesting asymptotics for singular values.

This minicourse is a pedagogical introduction to this model that presents the work of several authors. I will describe the Riemannian geometry found by Rauhut, Westdickenberg and their coauthors, as well as several features we have discovered in collaboration with Nadav Cohen, Florian Kogelbauer, Lulabel Ruiz-Seitz, and Zsolt Veraszto.

- **Eulalia Nualart (Universitat Pompeu Fabra)**

Title: *An introduction to the stochastic heat equation: local existence and blow-up.*

Abstract: In this course we will first define the concept of mild solution to the stochastic heat equation driven by a space-time white noise by recalling the theory of stochastic integrals with respect to Gaussian random fields. Then, under the assumption that the coefficients are locally Lipschitz functions, we will define and show local existence and uniqueness of the solution. Then, we will prove recent results that give necessary and sufficient conditions on the coefficients for the solution to blow-up in finite time, which are related to the well-known Osgood condition for ordinary differential equations.

Plenary Talks

- **Jorge Bolaños (Universidad Autónoma Metropolitana)**

Title: *A tour around circulant and G-circulant Quantum Markov Semigroups*

Abstract: The evolution of open quantum systems i.e., systems interacting with an external influence such as the environment or a bath, are modeled by the mathematical objects known as Quantum Markov Semigroups (QMS). In this talk i will review some of the problems of interest regarding QMS such as describing the structure of invariant states, the asymptotical behaviour and the spectral gap among others. I will use two special classes of semigroups as touchstone: circulant QMS and G-circulant QMS. The former class, first introduced a decade ago, inherits many of the simetries from the subalgebra of circulant matrices (and from the underlying group inducing them Zn) which have proven useful in the study of the aforementioned problems. The latter class however, which generalizes the first one to the case of arbitrary finite group G, gives new insight on how the underlying group induces , and to what extent, the useful simetries which help to tackle the problems of interest.

- **Saraí Hernández-Torres (UNAM)**

Title: *An Invitation to Uniform Spanning Trees.*

Abstract: Uniform spanning trees exhibit remarkable connections with various probability models, including simple random walks, random interlacements, the random cluster model, determinantal processes, and electrical networks. In this talk, we will provide a brief survey of these connections and recent achievements to emphasize the role that uniform spanning trees play in modern probability theory.

- **Adolfo Minjárez Sosa (Universidad de Sonora)**

Title: *Large population games*

Abstract: Large population games are characterized by involving a large number of objects. These can be the population to control (agents, data, particles, living beings, etc.) or the set of decision makers or controllers (players). The latter are the so-called mean field games. In this talk, we will provide an overview of the key concepts related to study of both class of games. Specifically, by applying the mean field theory we will describe the concepts of optimality and equilibrium, as well as the corresponding approximation schemes.

- **James Mingo (Queen's University)**

Title: *Infinitesimal Operators in Free Probability*

Abstract: In this talk (which is joint work with Pei-Lun Tseng (NYU Abu Dhabi)) I will show that free independence can be adapted to give spectral results on finite rank perturbations of unitarily invariant matrix ensembles.

The main concepts are that of an infinitesimal operator (the finite rank perturbation) and infinitesimal free independence. Free independence is Voiculescu's adaptation of independence to non-commuting random variables based on free products, and infinitesimal independence is a stronger form of free independence. Free independence has become in recent years one of the main tools in analyzing the eigenvalue distribution of sums and products of random matrices.

I will apply this to the commutator and anti-commutator of independent operators.

- **Mariana Olvera Cravioto(Chapel Hill)**

Title: *La evolución del opiniones en redes complejas: aproximaciones para redes densas y redes dispersas*

Abstract: Proponemos un modelo para la evolución de opiniones en una red compleja que explica nuestra realidad actual donde muchas cuestiones culturales exhiben polarización. Nuestro modelo es similar al popular modelo de Friedkin-Johnsen, con la complejidad adicional de que las señales emitidas por los medios de comunicación dependen de las características de cada individuo, reflejando sesgos cognitivos como nuestra propensidad a ver sólo lo que queremos ver. Este y otros sesgos nos permiten explicar los mecanismos más importantes que conducen a la polarización. Nuestro análisis está basado en un gráfico aleatorio dirigido, capaz de replicar las propiedades más importantes de las redes sociales, incluyendo su heterogéneidad, su grado de asortatividad y de modularidad. Nuestros teoremas establecen la distribución estacionaria de las opiniones en la red, e incluyen fórmulas explícitas para calcular las esperanzas y varianzas en cada comunidad de individuos. Nuestros resultados abarcan todo el rango de gráficos aleatorios heterogéneos, desde el régimen disperso, donde los grados esperados están acotados, hasta el régimen denso, donde un gráfico que tiene n vértices tiene un número de aristas de orden n^2.

- **Sandra Palau (UNAM)**

Title: *Fixation times for a multitype Lambda Wright-Fisher process.*

Abstract: We derive mixing and fixation times for the multi-type Lambda Wright-Fisher process with and without mutations. Our method relies on a grand coupling of the process realized through the so-called lookdown-construction.

- **Jose Luis Pérez Garmendia (CIMAT)**

Title: *Branching Processes, coalescents, and moment duality.*

Abstract: In this talk, we will review some recent results that provide the notion of a generalized ancestry for continuous-state branching processes with immigration by means of moment duality. This generalization includes cases such as the duality between mutation and death and the duality between pairwise branching and efficiency, and in the symmetric case, a relationship between branching processes and coalescents is uncovered. Finally, we will study the genealogy of asymmetric populations by introducing the partial order of adaptation and the asymmetric ancestral selection graph.

Thematic Sessions

Stochastic Control Theory

Organizer: Alejandra Fonseca Morales

- **Leonardo R. Laura-Guarachi (Escuela Superior de Economía - IPN)**

Title: *Optimal control problem of a success runs Markov chain*

Abstract: First, we review the structure and asymptotic properties of the well known "success-runs Markov chain" model. Then we consider it as a (deterministic) linear control system in a space of probability measures, which allows us to characterize a family of "normal stationary states". Then, considering the clas- sical ergodic optimality criteria (average optimality, good optimality, etc.), we describe the "optimal steady state" and the behavior of the time-dependent op- timal solution in the long term. To illustrate the theoretical results, we consider a forestry management problem in which the presence of the age mortality rate is not negligible (possibly caused by extreme weather events such as drought, wildfires, insect pests, etc.), and determine the "maximum sustainable yield" or "golden-rule forest".

- **Erick Treviño Aguilar (IMATE -UNAM)**

Title: *Interchange rules in stochastic optimization*

Abstract: In this talk we characterize conjugates and subdifferentials of convex integral functionals over linear spaces of cadlag stochastic processes. The approach is based on a new interchange rule of integral functional. The main result provide a general approach to apply convex duality in a variety of optimization problems. Joint work with Ari-Pekka Perkkio.

- **Daniel Hernández (CIMAT)**

Title: *Optimización de portafolios con restricciones trayectoriales*

Abstract: En esta plática consideraremos un problema de maximización de la utilidad para un agente que tiene algunas creencias de modelo, según las cuales el agente tratará de maximizar su utilidad, aunadas a restricciones que se basan en con- sideraciones independientes del modelo. La idea básica es que, suponiendo que el agente sólo observa trayectorias "posibles" según sus creencias, perseguir á un objetivo de maximización de la utilidad, pero si sus pérdidas alcanzan un nivel inaceptable (por ejemplo, debido a un comportamiento en el mercado fuera de modelo), deber á ser capaz de cumplir una restricción presupuestaria en todos los modelos posibles. Bajo estos supuestos de modelación, nuestro objetivo ser á determinar la estrategia de inversión óptima del agente cuando pueda tomar posiciones (estáticas) en determinadas opciones, por ejemplo, canastas de opciones de compra u otros derivados simples.

Stochastic Analysis

Organizer: Arturo Jaramillo (CIMAT)

- **Francisco Delgado Vences (UNAM)**

Title: *Inference for a stochastic partial differential equation related to an ecological niche*

Abstract: In this talk, we use a stochastic partial differential equation (SPDE) as a model for the density of a population. Indeed, we are interested in modeling animal density under the influence of random external forces/stimuli given by the environment. We want to study statistical properties for two crucial parameters of the SPDE that describe the dynamic of the system. To do that we use the Galerkin projection to transform the problem, passing from the SPDE to a system of independent SDEs; in this manner, we are able to find the Maximum likelihood estimator of the parameters. We validate the method by using simulations of the SPDE. We show consistency and asymptotic normality of the estimators; the latter is proved using the Malliavin-Stein method. These will allow us to fit the model to actual data.

- **Sefika Kuzgun (University of Rochester)**

Title: *Convergence of densities for stochastic heat equation*

Abstract: Let u be the solution to the one-dimensional stochastic heat equation. The purpose of this talk is to present the results on the uniform convergence of the density of the normalized spatial averages of the solution u over an interval $[-R, R]$, as R tends to infinity. These results are based on the combination of Stein method for normal approximations and Malliavin calculus techniques. This talk is based on joint works with David Nualart.

- **Chiara Amorino (Universitat Pompeu Fabra)**

Title: *Quantitative and stable limits of high-frequency statistics of Levy processes: a Stein's method approach*

Abstract: We establish inequalities for assessing the distance between the distribution of errors of partially observed high-frequency statistics of multidimensional Lévy processes and that of a mixed Gaussian random variable. Furthermore, we provide a general result guaranteeing stable convergence. Our arguments rely on a suitable adaptation of the Stein's method perspective to the context of mixed Gaussian distributions, specifically tailored to the framework of high-frequency statistics.

Branching and Duality

Organizer: Lizbeth Peñaloza Velasco (UMAR)

- **Alejandro Hernández Wences**

Title: *Lamperti Transforms of Self-Similar Measure-Valued Processes and Simple Coalescents*

Abstract: In this presentation, I will share our collaborative work with Arno Siri-Jégousse in which we derived a Lamperti transform for self-similar processes that take values in normed vector spaces. This transformation involves a random time change followed by a "(log) polar" decomposition of the state space. The resulting process is a Markov additive process (MAP) in which the '(log) norm' coordinate is additive-homogeneous. Lamperti originally studied the case of self-similar processes taking values in the positive reals, resulting in a MAP that is essentially a Lévy process. Alili et al. in 2017 extended his work to processes taking values in R^d, where the 'argument' coordinate now becomes non-trivial. We will demonstrate an application of our generalization to processes taking values in the space of positive measures, thereby expanding upon the results of Birkner et al. in 2005. They showed that for self-similar measure-valued branching processes, the time-changed and renormalized process (i.e., the 'argument' in our setting) is the Fleming-Viot process, which is in duality with the Beta Coalescent. We strengthen this result and obtain Fleming-Viot processes that are in duality with general Lambda-Coalescents

- **Adrián González Casanova UNAM, UC Berkeley)**

Title: *Sample duality*

Abstract: Heuristically, two processes are dual if one can find a function to study one process by using the other. Sampling duality is a duality which uses a duality function $S(n, x)$ of the form "what is the probability that all the members of a sample of size n are of a certain type, given that the number (or frequency) of that type of individuals is x". Implicitly, this technique can be traced back to the work of Blaise Pascal. Explicitly, it was studied in a paper of Martin Möhle in 1999 in the context of population genetics. We will discuss examples for which this technique is useful, including an application to the Simple Exclusion Process with reservoirs. The last part of the lecture is based on recent joint work with Simone Floriani https://arxiv.org/abs/2307.02481

- **Osvaldo Angtuncio Hernández (UMAR)**

Title: *Convergence of the Aldous-Broder process on high-dimensional graphs*

Abstract: The Aldous-Broder process on a graph $G = (V, E)$ is a process with values in the space of rooted trees whose vertex set is a subset of V which is stationary under the uniform distribution on the space of rooted trees spanning G. In Evans, Pitman and Winter (2006) the so-called root growth with regrafting process (RGRG) was constructed, and it was proved that its stationary distribution in the so called Continuum Random Tree (CRT). Further it was shown that the suitable rescaled Aldous-Broder process on the complete graph converges to the RGRG weakly with respect to the Gromov-Hausdorff topology. On the other hand, it has been shown by Aldous that the stationary limits of the previous processes agree: the scaling limit of the uniform spanning tree on the complete graph is the CRT. This result was extended

by Peres and Revelle (2005), proving that (up to a dimension depending constant factor) the continuum random tree is with respect to the Gromov-weak topology the scaling limit of the uniform spanning tree on the torus $\mathbb{Z}_N^d$, $d \geq 5$. In the present talk we discuss that the suitable rescaled Aldous-Broder process on graphs satisfying some general assumptions, converges to the RGRG weakly with respect to the Gromov-Hausdorff topology when initially started in the trivial rooted tree.

Mathematical Statistics

Organizer: Maria Fernanda Gil-Leyva Villa (UNAM)

- **Mario Díaz Torres (UNAM)**

Title: *Analysis of Private Machine Learning Models via Couplings and Contraction*

Abstract: Machine learning models, with their remarkable memorization capabilities, raise significant privacy concerns, making privacy an integral consideration in their design and training processes. Amidst the diverse array of privacy-preserving training algorithms, DP-SGD stands out as a widely adopted choice. Nonetheless, despite its prevalence, establishing the theoretical guarantees of DP-SGD has been feasible only within the confines of stringent convexity and differentiability assumptions. In this presentation, we introduce a novel methodology rooted in Markov chain couplings and contraction coefficients of Markov kernels, enabling us to establish theoretical guarantees for DP-SGD under minimal assumption.

- **Alan Riva Palacio Cohen (UNAM)**

Title: *Análisis de sobrevivencia Bayesiano no paramétrico con métodos de martingala.*

Abstract: En esta plática discutiremos el uso de martingalas para motivar estimadores en el análisis de sobrevivencia. Tal enfoque es clásico en el contexto de inferencia frecuentista donde ha sido empleado en las propuestas de los estimadores de Nelson-Aalen, para la función cumulativa de riesgo, y Kaplan-Meier, para la función de supervivencia. Más aún, en el ámbito de la inferencia Bayesiana el uso de martingalas es útil para el estudio de las propiedades asintóticas para modelos de sobrevida. En particular para los modelos neutrales por la derecha, donde pueden ser utilizados para demostrar el teorema de Bernstein von-Mises que establece la consistencia de la distribución posterior a una tasa equivalente a la consistencia del estimador frecuentista no paramétrico de Nelson-Aalen. Tales resultados pueden ser extendidos al contexto de modelos con covariables como el modelo neutral por la derecha para múltiples poblaciones parcialmente intercambiables, regresión de Cox y modelos con tasas de riesgo aceleradas.

- **Carlos Fidel Selva Ochoa (UNAM)**

Title: *The impact of clustering on equity investment strategies*

Abstract: Investment strategies often rely on historical correlations to determine portfolio allocations. This is the case of modern portfolio theory. However, these correlations are often unreliable, and models such as Black-Litterman have been developed to incorporate prior market expectations. Discretionary traders and financial analysts might make use of industry classification systems such as NAICS or GICS to limit their investments or analysis to companies in a specific sector or industry, which increases the comparability and universe correlation. However these classifications are based solely on fundamental factors and using them to define financial strategies may not be optimal. To assess this potential impact, the performance of momentum and mean-reversion strategies is back-tested using several clustering methods.

Free Probability

Organizer: Octavio Arizmendi (CIMAT)

- **Jiun-Chau Wang (University of Saskatchewan)**

Title: *Irregularity with freely infinitely divisible laws*

Abstract: We give a brief summary on the irregularity of additive and multiplicative free convolutions with freely infinitely divisible laws. These results come from the joint works with Hao-Wei Huang [1,2] and the one with Hari Bercovici and Ping Zhong [3].

[1] Hao-Wei Huang and J.-C. Wang, *Regularity results for free Lévy processes*, Adv. Math. **402** (2022).
[2] Hao-Wei Huang and J.-C. Wang, *Regularity results for free multiplicative Lévy processes*, preprint (2023).
[3] Hari Bercovici, J.-C. Wang, and Ping Zhong, *Superconvergence and regularity of densities in free probability*, Trans. of the Amer. Math. Soc. **376** (2023).

- **Noriyoshi Sakuma (Nagoya City University)**

Title: *The outlier problem viewed from non-commutative probability theory*

Abstract: The outlier problem stands as a captivating challenge within the topic of deforming random matrices. We present our research in collaboration with Benoit Collins, Felix Leid, Katsunori Fujie, and Takahiro Hasebe.
This talk explains a critical concept known as "cyclic monotone independence" and its associated matrix model. Additionally, we outline a methodology for calculating the fluctuations of these outliers.

- **Mauricio Salazar (ENSOG)**

Title: *Faster convergence in the Boolean and free central limit theorems*

Abstract: In this talk, we will see that, in the case of measures of bounded support, there is a faster convergence in the free and Boolean central limit theorems when the third cumulant is zero. Moreover, in the free case, the more vanishing cumulants there are, the faster the convergence is. This results has no counterpart in the classical central limit theorem.